

# Online Appendix to "Simple Regression Based Tests for Spatial Dependence"

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This appendix presents tables and graphs of additional simulation results not included in the paper.

## North-east modified rook matrix with 25% share in the north-east

In the paper, we choose a specification where the share of units located in the north-east is approximately 75%. To check the robustness of our results to this choice, we also run simulations with a 25% share of units located in the north-east. Table (1) and figure (1) show that the results do not change compared to the 75% case.

## $k$ ahead and $k$ behind weight matrix design

Here, we employ a " $k$  ahead and  $k$  behind" spatial weight matrix (see e.g. Kelejian and Prucha 1999, Kapoor, Kelejian and Prucha 2007). In this design, the  $i$ -th row of the weight matrix, where  $k < i < N - k$ , has nonzero elements in positions  $i - k, i - (k - 1), \dots, i + (k - 1), i + k$ , directly relating each element of the matrix to the  $k$  immediate neighbors ahead and behind. Adjusting the first and last  $k$  rows appropriately creates a circular world. To check whether  $W_1^n \neq W_2^n$  changes our results, we set  $k = 5$  for  $W_1^n$  and  $k = 3$  for  $W_2^n$ . Following common practice in empirical applications, we row normalize the spatial weight matrix, yielding nonzero entries of  $1/(2k)$ . As reported in table (2) and figure (2), the results are fairly robust to the above modifications.

## Anselin(1988) Columbus, Ohio weight matrix

As an example of a real weight matrix, we consider the weight matrix from Anselin's (1988) Columbus, Ohio, crime dataset. Again, the results do not change much qualitatively (see table (3) and figure (3)).

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## References

- Anselin, L.: 1988, *Spatial Econometrics: Methods and Models*, Kluwer, Dordrecht.
- Kapoor, M., Kelejian, H. H. and Prucha, I. R.: 2007, Panel data models with spatially correlated error components, *Journal of Econometrics* **140**, 97–130.
- Kelejian, H. H. and Prucha, I. R.: 1999, A Generalized Moments Estimator for the Autoregressive Parameter in a Spatial Model, *International Economic Review* **40**(2), 509–533.

**Table 1:** Empirical Sizes, 5% level

n	$LM^a$	$\widetilde{LM}^a$	$LM^b$	$\widetilde{LM}^b$	$LM^c$	$\widetilde{LM}^c$
Homoskedasticity						
97	0.053	0.056	0.055	0.056	0.058	0.056
177	0.049	0.054	0.049	0.053	0.052	0.049
281	0.053	0.053	0.051	0.054	0.047	0.046
485	0.053	0.052	0.055	0.053	0.047	0.050
945	0.052	0.053	0.049	0.051	0.054	0.052
Heteroskedasticity						
97	0.150	0.042	0.168	0.050	0.339	0.050
177	0.184	0.050	0.193	0.051	0.396	0.041
281	0.201	0.048	0.204	0.048	0.422	0.047
485	0.234	0.048	0.235	0.050	0.432	0.046
945	0.238	0.049	0.236	0.048	0.449	0.041

*Note:* Empirical sizes are calculated using 5000 replications. *North-east modified rook matrix* with a share of 25% of the units located in the north-east.

**Table 2:** Empirical Sizes, 5% level

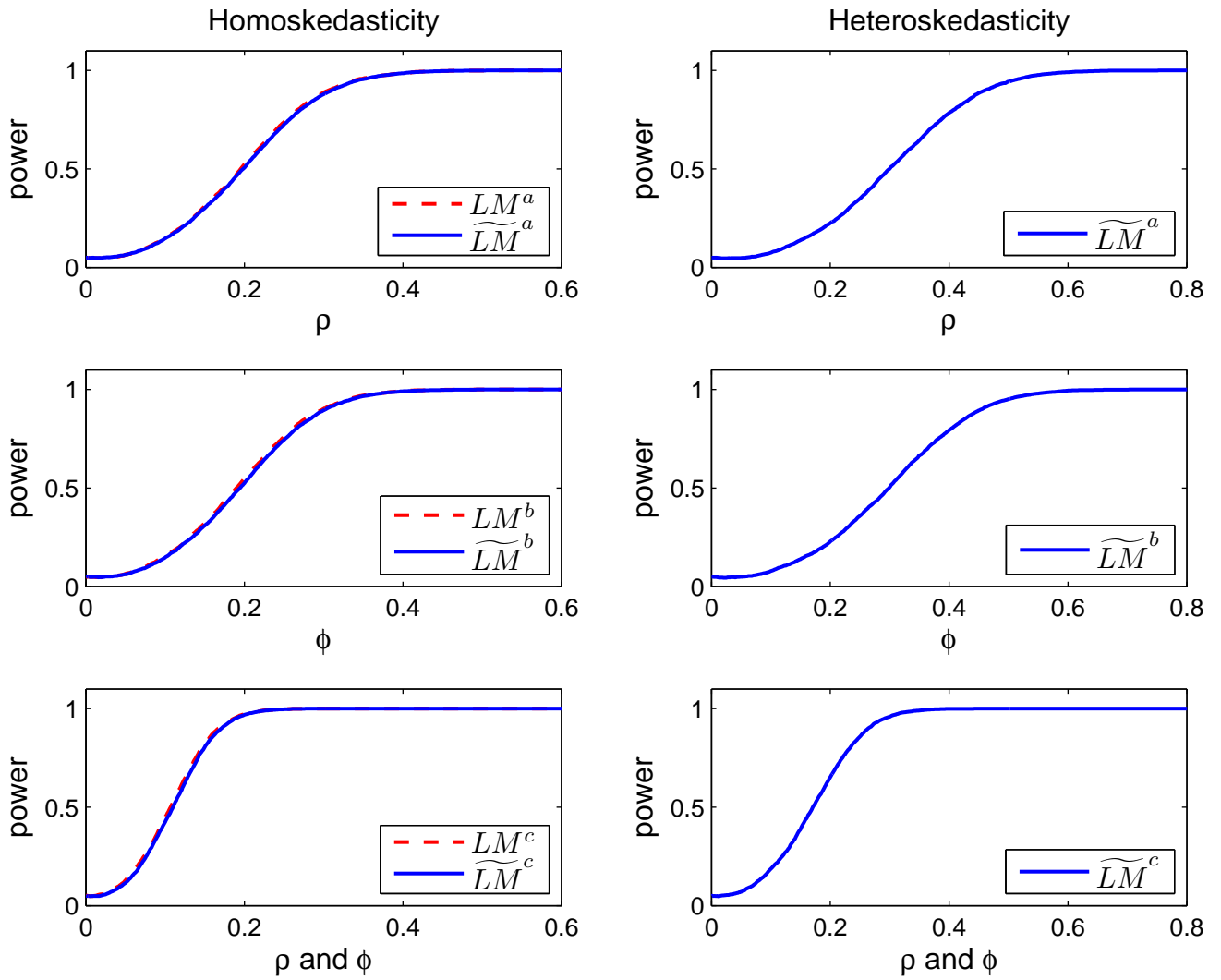
n	$LM^a$	$\widetilde{LM}^a$	$LM^b$	$\widetilde{LM}^b$	$LM^c$	$\widetilde{LM}^c$
Homoskedasticity						
100	0.034	0.062	0.043	0.054	0.038	0.049
150	0.042	0.061	0.048	0.058	0.044	0.047
250	0.043	0.057	0.050	0.052	0.050	0.040
500	0.045	0.055	0.050	0.052	0.050	0.036
1000	0.045	0.051	0.048	0.049	0.045	0.036
Heteroskedasticity						
100	0.138	0.045	0.283	0.044	0.348	0.040
150	0.164	0.046	0.290	0.043	0.371	0.039
250	0.203	0.049	0.290	0.047	0.389	0.037
500	0.220	0.051	0.282	0.049	0.400	0.037
1000	0.234	0.051	0.284	0.051	0.413	0.034

*Note:* Empirical sizes are calculated using 5000 replications. " $k$  ahead and  $k$  behind" specification.

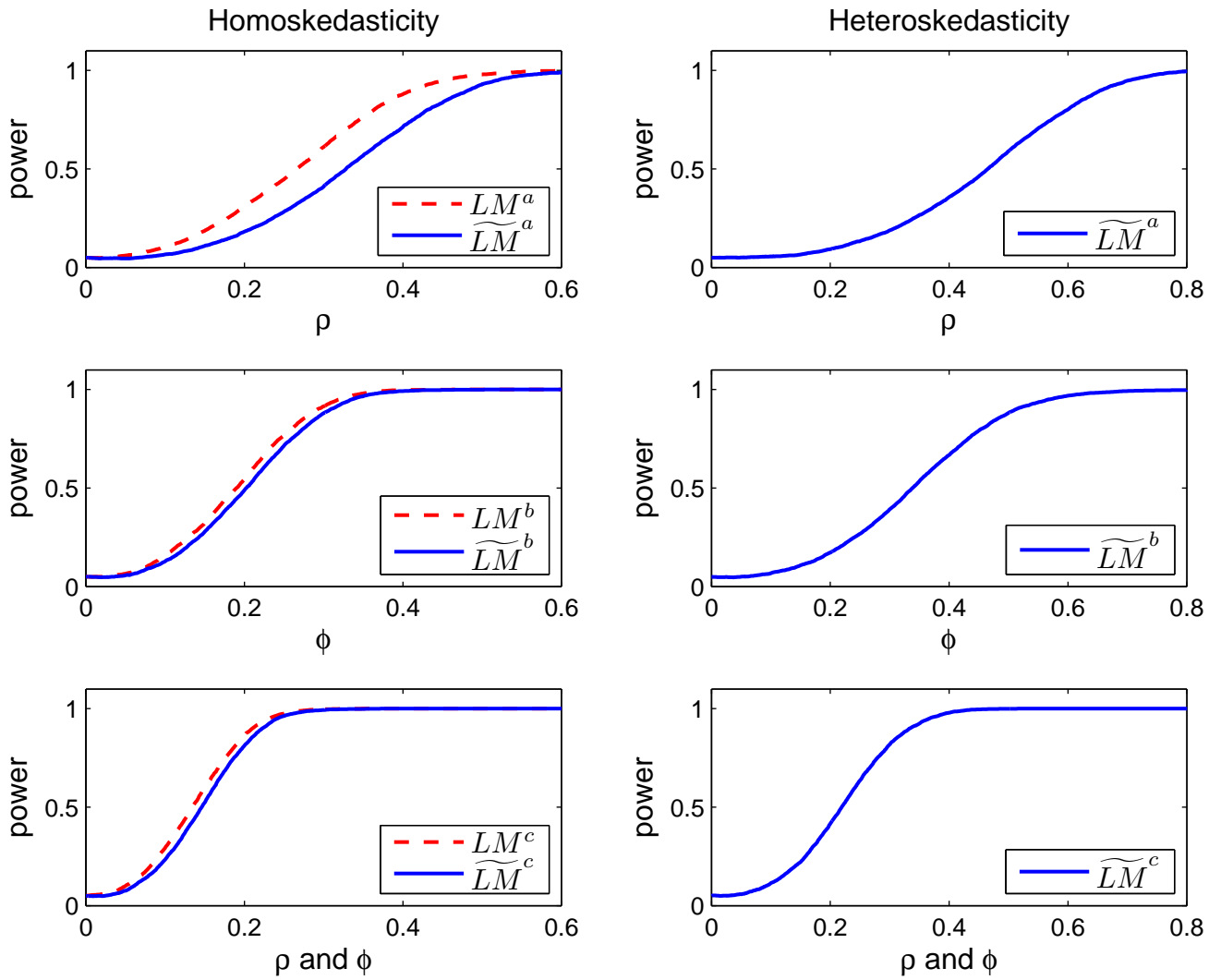
**Table 3:** Empirical Sizes, 5% level

n	$LM^a$	$\widetilde{LM}^a$	$LM^b$	$\widetilde{LM}^b$	$LM^c$	$\widetilde{LM}^c$
Homoskedasticity						
49	0.046	0.061	0.051	0.059	0.048	0.049
Heteroskedasticity						
49	0.179	0.041	0.150	0.042	0.168	0.035

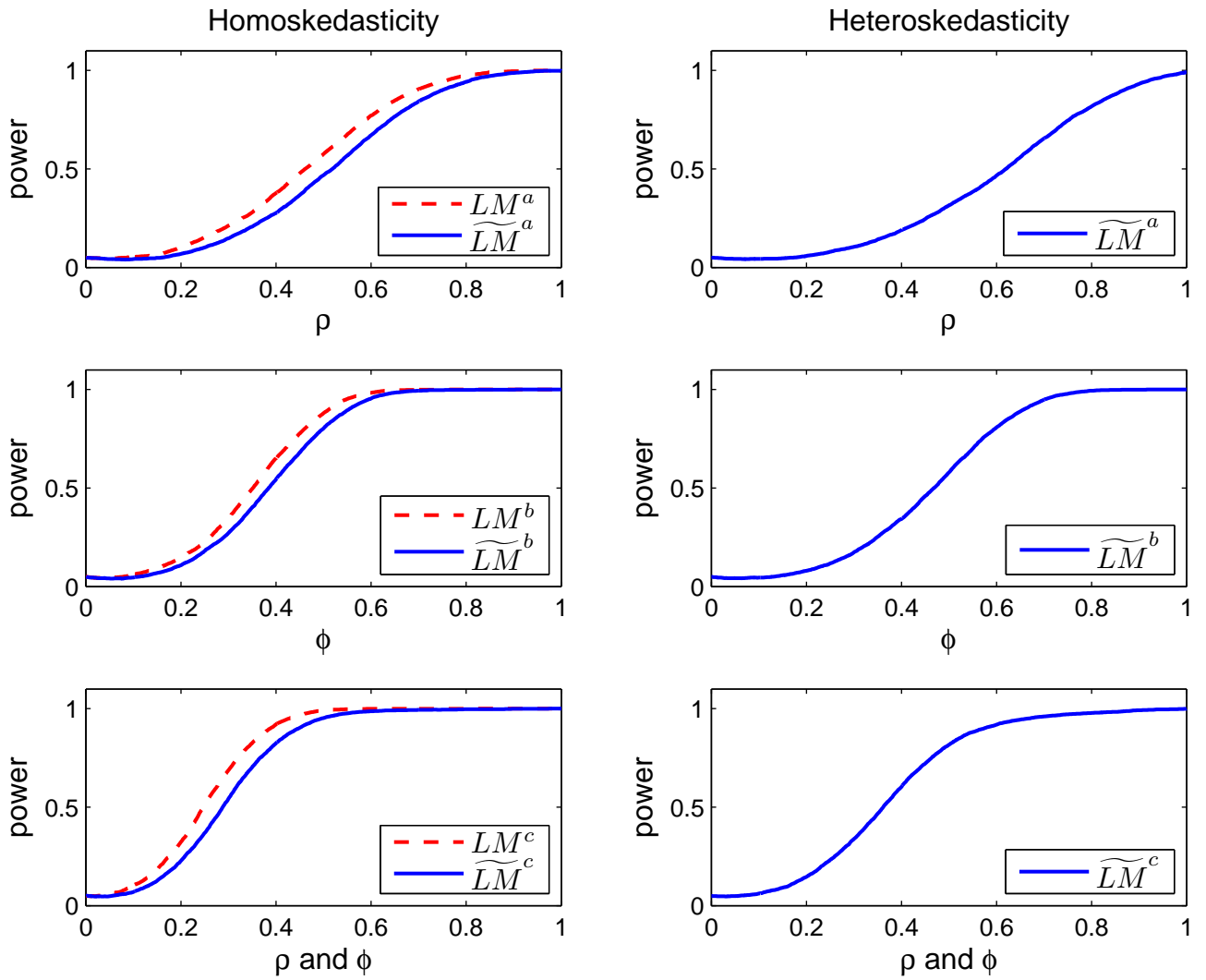
*Note:* Empirical sizes are calculated using 5000 replications. Columbus, Ohio grid used by Anselin (1988).



**Figure 1:** Size corrected power under homo- and heteroskedasticity ( $n=281$ ). *North-east modified rook matrix* with a share of 25% of the units located in the north-east.



**Figure 2:** Size corrected power under homo- and heteroskedasticity ( $n=250$ ). " $k$  ahead and  $k$  behind" specification.



**Figure 3:** Size corrected power under homo- and heteroskedasticity ( $n=49$ ). Columbus, Ohio grid used by Anselin (1988).